

On Hpp-wave/CFT₂ Holography ¹

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Abstract

We briefly review the AdS_3/CFT_2 correspondence and the holographic issues that arise in the Penrose limit. Exploiting current algebra techniques, developed by D’Appollonio and Kiritsis for the closely related Nappi-Witten model, we obtain preliminary results for bosonic string amplitudes in the resulting Hpp-wave background and comment on how to extend them to the superstring.

The AdS_3/CFT_2 holographic correspondence relates superstring theory on $AdS_3 \times S^3 \times \mathcal{M}$ to a two-dimensional superconformal field theory defined as the non-linear σ -model whose target space is the symmetric orbifold $\mathcal{M}^N/\mathcal{S}_N$ [1, 2, 3]. The four dimensional compactification manifold, \mathcal{M} , is chosen to be T^4 or $K3$.

$AdS_3 \times S^3 \times \mathcal{M}$ can be thought of as the near horizon geometry of a D1-D5 brane configuration with non-vanishing R-R 3-form flux. Since it is not fully known how to quantize the superstring in the presence of generic R-R backgrounds², it is convenient to consider the S-dual configuration which is supported by a NS-NS 3-form flux [3]. The metric of the resulting F1-NS5 bound state is

$$ds^2 = f_1^{-1} (-dx_0^2 + dx_1^2) + f_5 (dr^2 + r^2 d\Omega_3^2) + ds_{\mathcal{M}}^2,$$

$$f_1 = 1 + \frac{g_s^2 \alpha' Q_1}{vr^2}, \quad f_5 = 1 + \frac{\alpha' Q_5}{r^2},$$

where f_1, f_5 are harmonic functions in the transverse space, Q_1, Q_5 are the number of fundamental strings and 5-branes, respectively, and v is the volume of \mathcal{M} in string units. $AdS_3 \times S^3 \times \mathcal{M}$ (in Poincaré coordinates) emerges in the near horizon limit $r \rightarrow 0$ after the change of variables $g_s^2 = v Q_5/Q_1$ and $r = R^2/u = Q_5 \alpha'/u$.

In principle, one can consider $AdS_3 \times S^3$ supported by both R-R and NS-NS 3-form fluxes, where the supergravity equations give

$$R^2 = \sqrt{(Q_5^{NS})^2 + g_s^2 (Q_5^R)^2}.$$

The six-dimensional hybrid formalism of Berkovits-Vafa-Witten (BVW) [6] is suitable for quantization in these backgrounds and allows to compute some supersymmetric amplitudes [7].

Since the bosonic string provides useful insights into some basic aspects of holography, we will first concentrate on this simpler yet interesting case [3, 4]. The bosonic string on $AdS_3 \times S^3$ can be described by a Wess-Zumino-Novikov-Witten (WZNW) model with left and right affine algebras $\widehat{SL}(2, \mathbf{R}) \times \widehat{SU}(2)$, which is a completely solvable conformal field theory. Denoting by $K^A(z)$ the generators of $\widehat{SL}(2, \mathbf{R})$ and those of $\widehat{SU}(2)$ by $J^a(z)$, we can write the current algebras as follows

$$\begin{aligned} \widehat{SL}(2, \mathbf{R}) : \quad K^A(z) K^B(w) &\sim \frac{k \eta^{AB}}{2(z-w)^2} + i \epsilon^{ABC} \frac{K^C(w)}{z-w}, \\ \widehat{SU}(2) : \quad J^a(z) J^b(w) &\sim \frac{k \delta^{ab}}{2(z-w)^2} + i \epsilon^{abc} \frac{J^c(w)}{z-w}, \end{aligned}$$

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²At present, the Pure Spinor formalism is the most promising approach to tackle this outstanding problem [5].

where the Cartan Killing metric for $\widehat{SL}(2, \mathbf{R})$ is chosen to be $\eta^{AB} = \text{diag}(+, +, -)$. Here we are considering the case $k_{SU(2)} + 2 = k_{SL(2)} - 2 \gg 1$ and $c_{int} = 20$.

The problem of quantizing the bosonic string in backgrounds containing an AdS_3 factor has a long story [8]. Only recently it has become clear that the spectrum consists not only of short strings corresponding to “discrete” representations with $1/2 < j < (k-1)/2$ and long strings corresponding to “continuous” representations with $j = 1/2 + is$, but also of spectral flowed representations characterized by an integer w , to some extent similar to a winding number [4]. String amplitudes with a low number of insertions (two, three and four) can be computed via current algebra methods or the Wakimoto free field representation [4, 9, 10]. Extension to the superstring should not present any major obstacle and we plan to address this issue in the near future [11]. Yet quantitative comparison with boundary conformal field theory predictions is hampered by the presence of moduli deformations of the non-linear σ -model [4, 12] which would be lifted by turning on a R-R background.

As mentioned above this is not fully under control, except possibly for the Hpp-wave which is the local geometry seen by an observer moving at the speed of light in $AdS_3 \times S^3$. This process of zooming-in around a null geodesic is called Penrose limit [13, 14].

In global coordinates, $AdS_3 \times S^3$ has the form

$$ds^2 = R^2 [-(\cosh \rho)^2 dt^2 + d\rho^2 + (\sinh \rho)^2 d\varphi_1^2 + (\cos \theta)^2 d\psi^2 + d\theta^2 + (\sin \theta)^2 d\varphi_2^2],$$

where (t, ρ, φ_1) are the coordinates on AdS_3 and $(\theta, \psi, \varphi_2)$ are those on S^3 . In order to perform the Penrose limit, we change variables to

$$t = \frac{u}{2} + \frac{v}{R^2}, \quad \psi = \frac{u}{2} - \frac{v}{R^2}, \quad \rho = \frac{r_1}{R}, \quad \theta = \frac{r_2}{R},$$

and take the limit of infinitely large radius for both spaces. A useful change of variables is $r_i \exp(i\varphi_i) = \exp(iu/2)w_i$.

We note here that an unexpected $SU(2)_I$ symmetry comes from the fact that the symmetry of the 4-d transverse plane is broken by the NS-NS 3-form flux $H_{+12} = H_{+34} = 1$ according to $SO(4) \rightarrow SU(2)_I \times U(1)_J$ ³. Taking into account this extra symmetry, the Hpp-wave metric takes the manifest $SU(2)_I$ invariant form

$$ds^2 = -2dudv + \frac{i}{4}du \sum_{\alpha=1}^2 (w^\alpha d\bar{w}_\alpha - \bar{w}_\alpha dw^\alpha) + \sum_{\alpha=1}^2 dw^\alpha d\bar{w}_\alpha.$$

As usual we can introduce momenta along the light-cone directions, $j \equiv -i \partial_u$ and $p \equiv -i \partial_v$.

In line with what we said above we expect string theory in this pp-wave background to be described by a special kind of WZNW model. In particular, for the Penrose limit of $AdS_3 \times S^3$, the worldsheet CFT is a generalization of the Nappi-Witten (NW) model, which in turn emerges from the Penrose limit of the near horizon geometry of a stack of NS5-branes [16]. In our case the Heisenberg algebra is six-dimensional (\mathcal{H}_6), while for the standard NW model it is four-dimensional (\mathcal{H}_4). From the current algebra point of view the Penrose limit is carried out by contracting the currents of both CFTs as

$$K(z) = \frac{i}{k} [J^3(z) - K^3(z)], \quad J(z) = -i [J^3(z) + K^3(z)],$$

and taking the limit $k \rightarrow \infty$. The other generators are just scaled by the constant factor $\sqrt{2/k}$. We obtain a Heisenberg algebra

$$[P_\alpha^+, P^{-\beta}] = -2i\delta_\alpha^\beta K, \\ [J, P_\alpha^+] = -iP_\alpha^+, \quad [J, P^{-\alpha}] = +iP^{-\alpha}.$$

³ $H_{+12} = \mu_1 \neq H_{+34} = \mu_2$ gives an exactly solvable CFT on the string worldsheet, too. We will not discuss the general case here (see [18]).

It should be stressed that current contraction (à la Saletan) is a general procedure that can be applied to other models [15], *e.g.* to the superstring in the already mentioned BVW approach ⁴.

Exploiting current algebra techniques developed for the NW model by D’Appollonio and Kiritsis [16], or the alternative free field Wakimoto representation proposed by Cheung, Freidel and Savvidy [17], one can derive explicit expressions for some string amplitudes in the Hpp-wave background. Here we only report some preliminary results. An extensive analysis will be presented in a forthcoming paper [18].

First of all recall that \mathcal{H}_6 has three types of representations, depending on the value of the light cone momentum p . States can have $p \neq 0$ (discrete representations) or $p = 0$ (continuous representations). The primary fields that we can construct are of the general form $\Phi_q^a(z, \bar{z}; x, \bar{x})$, where a specifies the type of representation, $a = \pm, 0$ for $p \geq 0$ or $p = 0$, respectively, q stands for the momenta, $q = (p, \hat{j})$ for $p \neq 0$ and $q = (s, \hat{j})$ for $p = 0$, and x , with the appropriate index (x_α for $p \geq 0$ and x^α for $p < 0$), are some auxiliary “charge” variables that compactly encode all the states in a given representation. Note that in this covariant approach we naturally include $p = 0$ states, inaccessible to the light cone quantization. Using the charge variables, $\widehat{\mathcal{H}}_6$ can be realized in terms of differential operators

$$\begin{aligned} p > 0 : \quad & P_\alpha^+ = \sqrt{2} p x_\alpha, \quad P^{-\alpha} = \sqrt{2} \partial^\alpha, \quad J = i (\hat{j} + x_\alpha \partial^\alpha), \quad K = ip, \\ p < 0 : \quad & P_\alpha^+ = \sqrt{2} \partial_\alpha, \quad P^{-\alpha} = \sqrt{2} p x^\alpha, \quad J = i (\hat{j} - x^\alpha \partial_\alpha), \quad K = -ip. \end{aligned}$$

From now on we use the notation $p \equiv |p|$ for states with $p < 0$.

With this realization of the algebra the two-point function is easily obtained by imposing the Ward identities

$$\langle \Phi_{p_1, \hat{j}_1}^+(z_1, \bar{z}_1, x_{1\alpha}, \bar{x}_1^\alpha) \Phi_{p_2, \hat{j}_2}^-(z_2, \bar{z}_2, y_2^\alpha, \bar{y}_{2\alpha}) \rangle = \delta(p_1 - p_2) \delta(\hat{j}_1 + \hat{j}_2) \prod_{\alpha=1}^2 \frac{e^{-p_1(x_{1\alpha} x_2^\alpha + \bar{x}_1^\alpha \bar{y}_{2\alpha})}}{|z_{12}|^{4h}},$$

where $h_\pm = \mp p \hat{j} + p(1 - p)$ and the $SU(2)_I$ symmetry is manifest.

By the same procedure the kinematical x dependent part of the three-point function between two ‘incoming’ $p > 0$ states and one ‘outgoing’ state with $p < 0$ (by momentum conservation $\delta(p_1 + p_2 - p_3)$), is given by

$$K_{++-}(x_{1\alpha}, x_{2\alpha}, x_3^\alpha; \bar{x}_1^\alpha, \bar{x}_2^\alpha, \bar{x}_{3\alpha}) = \prod_{\alpha=1}^2 |e^{-x_3^\alpha(p_1 x_{1\alpha} + p_2 x_{2\alpha})}|^2 (x_{2\alpha} - x_{1\alpha})^{q_\alpha} (\bar{x}_2^\alpha - \bar{x}_1^\alpha)^{q_\alpha},$$

where $L = -(\hat{j}_1 + \hat{j}_2 + \hat{j}_3) = \sum_\alpha q_\alpha$ and $q_1, q_2 \in \mathbf{N}$. A sum over q_1 and q_2 is implicit. The important point we would like to stress here is that in order to obtain an $SU(2)_I$ invariant expression we should put together left and right moving parts, since the relevant $SU(2)_I$ does not admit a chiral worldsheet description.

Ward identities also fix the kinematical x dependent part of the extremal four-point function $\langle +++- \rangle$. The left piece is

$$K_{++++}^{LEFT}(x_{1\alpha}, x_{2\alpha}, x_{3\alpha}, x_4^\alpha) = \prod_{\alpha=1}^2 e^{-x_4^\alpha(p_1 x_{1\alpha} + p_2 x_{2\alpha} + p_3 x_{3\alpha})} (x_{3\alpha} - x_{1\alpha})^{q_\alpha}.$$

Once again we have an expression that is not $SU(2)_I$ chiral invariant. The full correlator, including left and right movers, contains also the dynamical part

$$\prod_{\alpha} \frac{1}{q_\alpha!} (C_{12} \|f(z, x_{i\alpha})\|^2 + C_{34} \|g(z, x_{i\alpha})\|^2)^{q_\alpha},$$

where $f_\alpha(z, x_{i\alpha}) = \sum_{i=1}^2 (x_{i\alpha} - x_{3\alpha}) f_i(z)$ and $g_\alpha(z, x_{i\alpha}) = \sum_{i=1}^2 (x_{i\alpha} - x_{3\alpha}) g_i(z)$; $f_i(z)$ and $g_i(z)$ are given in terms of hypergeometric functions [16, 17, 18]. The coefficients C_{12} and C_{34} entering in this monodromy invariant combination are

$$C_{12} = \frac{\gamma(p_1 + p_2)}{\gamma(p_1)\gamma(p_2)}, \quad C_{34} = \frac{\gamma(p_4)}{\gamma(p_3)\gamma(p_1 + p_2)}.$$

⁴I thank N. Berkovits for pointing out this possibility.

Other correlators, in particular those with insertion of $p = 0$ states, can be treated in a similar though subtler way [18]. As expected, string amplitudes in the Hpp-wave can be obtained from those of $AdS_3 \times S^3$ by performing the Penrose limit on the $SL(2)$ and $SU(2)$ ‘charge variables’, along the lines of [16, 18]. We thus see the dual role of the ‘charge’ variables: as a compact bookkeeping of the field content of a given representation and as coordinate on a holographic screen [18, 23]. In order to attempt any quantitative test of this holographic interpretation one has to consider the superstring.

In a quasi-free field realization, the bosonic generators of the supersymmetric version of \mathcal{H}_6 algebra are [24]

$$K = \oint \frac{dz}{2\pi i} \partial v(z), \quad J = \oint \frac{dz}{2\pi i} \partial u(z),$$

$$P_\alpha^+ = \oint \frac{dz}{2\pi i} e^{-iu} (i\partial w_\alpha^* + \psi^+ \psi_\alpha^*)(z), \quad P^{-\alpha} = \oint dz e^{iu} (i\partial w^\alpha + \psi^- \psi^\alpha)(z),$$

where the worldsheet fields contract according to

$$u(z)v(w) \sim \log(z-w), \quad w_\alpha^*(z)w^\beta(w) \sim -\delta_\alpha^\beta \log(z-w),$$

$$\psi^+(z)\psi^-(w) \sim 1/(z-w), \quad \psi_\alpha^*\psi^\beta(w) \sim \delta_\alpha^\beta/(z-w).$$

Depending on the choice of the internal manifold \mathcal{M} , super- \mathcal{H}_6 algebra will have 12 (for T^4) or 8 (for $K3$) fermionic generators in the left-moving sector (as many in the right-moving sector). This can be seen decomposing the left-moving spin field S^A [25] belonging to the **16** of $SO(9,1)$ into $(\mathbf{2}_L, \mathbf{2}_L)_+ + (\mathbf{2}_R, \mathbf{2}_L)_- + (\mathbf{2}_L, \mathbf{2}_R)_- + (\mathbf{2}_R, \mathbf{2}_R)_+$ of $SO(1,1) \times SO(4) \times SO(4)$. On the other hand BRST invariance and GSO projection suggest the following form for the four dynamical supercharges in the $(\mathbf{2}_R, \mathbf{2}_L)_-$

$$\mathcal{Q}_{dyn}^{-\dot{\alpha}a} = \oint \frac{dz}{2\pi i} e^{-\phi/2} S^{-\dot{\alpha}a}.$$

Similarly the four kinematical supercharges in the $(\mathbf{2}_L, \mathbf{2}_L)_+$ read

$$\mathcal{Q}_{kin}^{+\alpha a} = \oint \frac{dz}{2\pi i} e^{-\phi/2} S^{+\alpha a} e^{i\alpha u}.$$

These supersymmetries, common to both T^4 and $K3$, are in a sense the same supersymmetries the superstring already had on $AdS_3 \times S^3 \times \mathcal{M}$. However when taking the Penrose limit of $AdS_3 \times S^3 \times T^4$, the BRST condition relaxes and four extra supersymmetries in the $(\mathbf{2}_R, \mathbf{2}_R)_+$ arise ⁵

$$\mathcal{Q}_{new}^{+\dot{\alpha}a} = \oint \frac{dz}{2\pi i} e^{-\phi/2} S^{+\dot{\alpha}a}.$$

In the BVW formalism only 8(+8) supersymmetries are manifest, in agreement with the number of Green-Schwarz-like independent spacetime odd variables. This works out right for $K3$ [26]. However, for T^4 , the Penrose limit increases to 12(+12) the number of supersymmetries, making it impossible to have all of them explicitly manifest in the hybrid formalism. It seems that the 12(+12) manifest supersymmetries could only be obtained by starting from the pure spinor formalism [5], something that has not yet been done.

Once the explicit form of the supercharges is known, one can determine the supermultiplet structure and identify BPS states. The candidate vertex operators for 1/2 BPS states read [24]

$$\mathcal{V}_{BPS}^{(-),\alpha} = e^{-\phi} e^{ipv} \prod_i \sigma_p^{(i)} : \psi^\alpha \Sigma_p^{(i)} : ,$$

where $\sigma_p^{(i)}$ and $\Sigma_p^{(i)}$ are bosonic and fermionic twist fields. One can easily check that half of the supercharges annihilate $\mathcal{V}_{BPS}^{(-),\alpha}$, while the remaining half act on it and build up a short supermultiplet.

⁵For $K3$ the number of supersymmetries remain unchanged.

As mentioned earlier, the boundary CFT dual to superstrings on $AdS_3 \times S^3 \times \mathcal{M}$ is a non-linear σ -model on the symmetric orbifold $Sym^N(\mathcal{M}) = (\mathcal{M})^N / \mathcal{S}_N$, where \mathcal{S}_N is the symmetric group of $N = Q_1 Q_5$ elements. Following the BMN conjecture [27] that relates light-cone momenta in the pp-wave background to conformal dimensions and R-charges of the operators in the dual theory, the spectrum of the super-CFT was shown to be given by [28, 29]

$$\begin{aligned} \text{R-R:} \quad \Delta - J &= \sum_n N_n \sqrt{1 + \left(\frac{n g_s Q_5^R}{J} \right)^2} + g_s Q_5^R \frac{L_0^{\mathcal{M}} + \bar{L}_0^{\mathcal{M}}}{J}, \\ \text{NS-NS:} \quad \Delta - J &= \sum_n N_n \left(1 + \frac{n Q_5^{NS}}{J} \right) + Q_5^{NS} \frac{L_0^{\mathcal{M}} + \bar{L}_0^{\mathcal{M}}}{J}. \end{aligned}$$

where R-R and NS-NS stand for the nature of the 3-form flux.

At a first scrutiny, it seemed that the BMN correspondence failed to correctly match the spectra on the two sides even for states corresponding to operators with large R-charge [24]. Nevertheless, it has been suggested that, very much as for the original AdS_3/CFT_2 correspondence, this might be due to the fact that the ‘boundary CFT₂’ is sitting at the orbifold point, which is not the case for the bulk description. In principle one can dispose of this mismatch by a marginal deformation along the moduli space of the CFT₂ [30]. Alternatively one may, in principle, be able to extrapolate the string spectrum to the symmetric orbifold point, very much as in [19, 20, 21] for the case of AdS_5/CFT_4 , and find precise agreement [22].

In addition one may envisage the possibility of computing correlation functions in the ‘boundary theory’, which results from keeping only operators with large R-charge and where superconformal symmetry has been traded for (or more technically ‘contracted’ into) the relevant supersymmetric version of \mathcal{H}_6 . Eventually they should be expressed in terms of the very same charge variables that appear in the covariant string amplitudes in the Hpp-wave. They play the role of coordinates on a four-dimensional holographic screen, which is different from the geometric boundary of the Hpp-wave [31], but captures the essential features of the dynamics in the bulk [11].

Following [16, 23], we believe this is the way holography works in the pp-wave limit of spaces with AdS factors. Although we have only considered the simplest example of the Hpp-wave resulting from a Penrose limit of $AdS_3 \times S^3 \times \mathcal{M}$, we hope this may shed some light on the fate of holography in the more realistic case of $AdS_5 \times S^5$. In particular using a manifestly covariant and supersymmetric formalism, such the hybrid BVW [6] or the pure spinor formalism [5], one should be able to address the case with non-zero R-R 3-form flux [26] which represents the first hint to the case with R-R 5-form flux.

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